

Math 60 12.1 Distance and Midpoint Formulas - Day 2

Objective: 2) Use the midpoint formula to find

- The coordinates of an ordered pair which is halfway between two points.
- The coordinates of the midpoint of the line segment connecting two points.

3) Applications of the Distance and midpoint formulas.

① GOAL: Find the coordinates of the midpoint between $(5, -4)$ and $(3, 2)$.

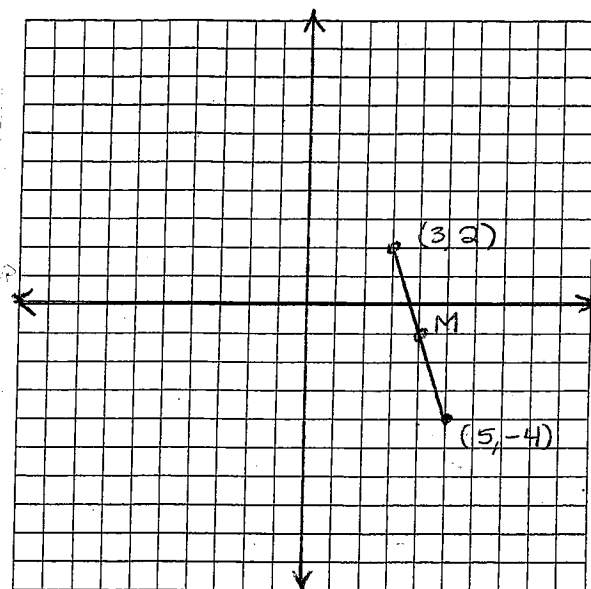
Long way:

Graph the two points.

Draw the line segment connecting them.

Draw the point which is halfway between.

Our goal is to find the coordinates (x, y) of this point.



The x-coordinate is halfway between the given x-coords:

x-coordinate = average of x-coordinates

$$x\text{-coord} = \frac{3+5}{2} = \frac{8}{2} = 4$$

y-coordinate = average of y-coordinates

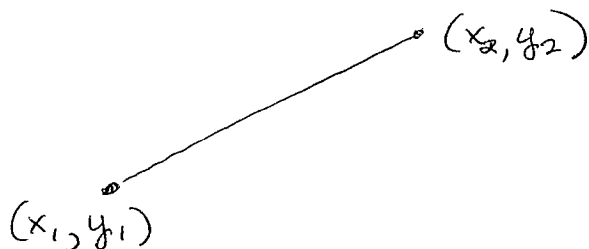
$$y\text{-coord} = \frac{2+(-4)}{2} = \frac{-2}{2} = -1$$

coordinates of midpoint $\boxed{(4, -1)}$

* Midpoint Formula: The coordinates of a point
 • halfway between (x_1, y_1) and (x_2, y_2) or
 • that is the midpoint of a line segment connecting (x_1, y_1) and (x_2, y_2)
 are given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$

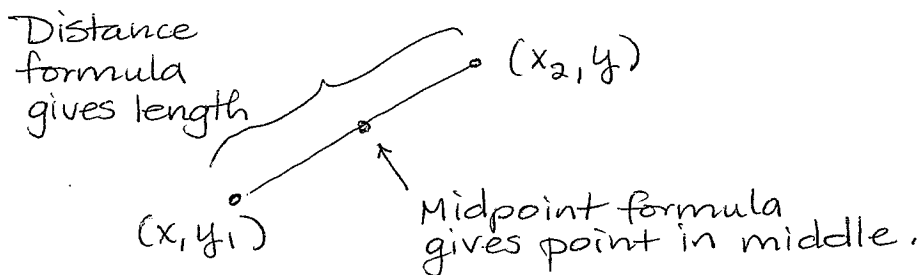
Memorize!

Ways the Distance and Midpoint Formulas are the same:



- They both start with a line segment connecting two points.
- They both include two ordered pairs called (x_1, y_1) and (x_2, y_2) in the instructions for the question.

Ways the Distance and Midpoint Formulas are different:



Distance formula has a square root. Midpoint does not.

Distance formula gives a single number for the result.

Midpoint formula gives an ordered pair for the result.

The distance formula is a single expression.

The midpoint formula is two expressions — one for the x-coordinate and one for the y-coordinate.

- ② Find the midpoint of the line segment formed by joining P_1 and P_2 .

$$P_1 = (2\sqrt{2}, \sqrt{5})$$

$$P_2 = (5\sqrt{2}, 4\sqrt{5})$$

$$\text{x-coord } \frac{2\sqrt{2} + 5\sqrt{2}}{2}$$

find average of the
x-coordinates

$$= \frac{7\sqrt{2}}{2}$$

$$\text{y-coord } \frac{\sqrt{5} + 4\sqrt{5}}{2}$$

find average of the
y-coordinates.

$$= \frac{5\sqrt{5}}{2}$$

$$\text{midpoint } \left(\frac{7\sqrt{2}}{2}, \frac{5\sqrt{5}}{2} \right)$$

- ③ Find all points having an x-coordinate of 4 whose distance from the point $(0,3)$ is 5.

"Find all points" $\Rightarrow$ want (x,y) .

"x-coordinate 4" $\Rightarrow$ want $(4,y)$

"distance" $\Rightarrow$ distance formula $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

"from the point $(0,3)$ " $\Rightarrow$ one of the points (x_1, y_1) or (x_2, y_2) is $(0,3)$.

$\Rightarrow$ the other point is $(4,y)$.

"distance is 5" $\Rightarrow$ distance formula = 5

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5$$

cont

$$(x_1, y_1) = (4, y)$$

$$(x_2, y_2) = (0, 3)$$

}

This is an arbitrary choice.
You can choose to swap
and still get the correct
answer.

Substitute:

$$\sqrt{(0-4)^2 + (3-y)^2} = 5$$

$$\sqrt{4^2 + (3-y)^2} = 5$$

$$(\sqrt{16 + (3-y)^2})^2 = (5)^2$$

$$16 + (3-y)^2 = 25$$

↑

simplify, $0-4=4$

simplify $4^2=16$

Square both sides

* caution *
might create extraneous
solution!

Recognize quadratic equation
containing a perfect square.

Isolate the perfect square.

$$(3-y)^2 = 25-16$$

$$(3-y)^2 = 9$$

$$3-y = \pm\sqrt{9}$$

$$3-y = \pm 3$$

$$3-y = 3$$

$$-y = 0$$

$$y = 0$$

$$3-y = -3$$

$$-y = -6$$

$$y = 6$$

Square root property

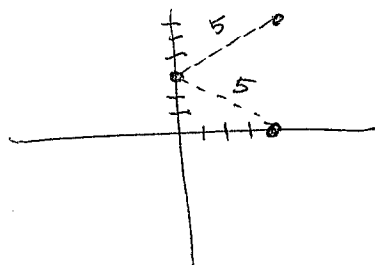
simplify $\sqrt{9}=3$

write two eqns

isolate y.

$$(4, 0) \text{ and } (4, 6)$$

are two points which are 5 units
away from $(0, 3)$ and have x-coord 4



- ④ Find all points having a y-coordinate -3 whose distance from the point $(-4, 2)$ is 13.

$$\begin{aligned} \text{Points } (x, -3) &= (x_1, y_1) \\ (-4, 2) &= (x_2, y_2) \end{aligned}$$

Distance formula = 13

$$\sqrt{(-4-x)^2 + (2+3)^2} = 13$$

$$\sqrt{(-4-x)^2 + 5^2} = 13$$

$$(\sqrt{(-4-x)^2 + 25})^2 = (13)^2$$

$$(-4-x)^2 + 25 = 169$$

$$(-4-x)^2 = 144$$

$$\sqrt{(-4-x)^2} = \pm\sqrt{144}$$

$$(-4-x) = \pm 12$$

$$\frac{-x}{-1} = \frac{4 \pm 12}{-1}$$

$$x = -4 \mp 12$$

$$x = -4 + 12, -4 - 12$$

- ⑤ Write an equation for the points (x, y) which are 3 units from the point $(-2, 5)$. What does this set of points mean?

1st point $(-2, 5)$

and point (x, y)

distance formula $\sqrt{(x-(-2))^2 + (y-5)^2} = 3$

$$\sqrt{(x+2)^2 + (y-5)^2} = 3$$

or $(x+2)^2 + (y-5)^2 = 9$

square both sides

This means a circle
distance 3 means
radius 3.

